MATH 417: Introduction to Abstract Algebra (3 credit hours)

Course description

Math 417 is an introduction to abstract algebra. The main objects of study are the fundamental theorem of arithmetic, congruences, permutations, groups, subgroups and homomorphisms. Additional topics include group actions with applications, polynomials, rings, subrings, ideals, integral domains and fields, roots of polynomials, maximal ideals and construction of fields.

Prerequisite: Either MATH 416 or one of MATH 410, MATH 415 together with one of MATH 347, MATH 348, CS 374; or consent of instructor.

Course Objectives

Students will begin by looking at binary operations and considering how they give rise to various algebraic structures, then move on to an in-depth study of groups and rings, including special cases of abelian groups, finite groups, domains, fields, PIDs, UFDs and more. Applications and examples of groups and rings will also be considered, including group actions, polynomials, matrices, and applications to number theory. In addition, students will prove various fundamental theorems and examine homomorphisms.

Course Content

1. Introduction to abstract algebra, groups and permutations

2. Order of group elements, parity of permutations, permutation matrices, algebraic properties of integers, greatest common divisor, lowest common multiple, well-ordering principle, congruence and modular arithmetic

3. Binomial theorem, Fermat's little theorem, cryptography, abstract groups and properties of groups, group isomorphisms, monoids and semigroups

4. Groups of small order, subgroups, generators of groups, cyclic subgroups, cyclic groups, group homeomorphisms

5. Normal subgroups, cosets, Lagrange's theorem, index of a subgroup

6. Even order theorem, generalized Lagrange theorem, equivalence relations, quotient groups, cycle conjugation of permutations and applications, homomorphism theorem for quotients, first isomorphism theorem and examples

7. Correspondence theorem for isomorphisms, factorization theorem for isomorphisms, product subsets, diamond isomorphism theorem, direct product of groups, Chinese remainder theorem

8. Automorphisms of groups, Inn(G) and Out(G), conjugation, center of a group, semidirect products, identification theorems for direct and semidirect products

9. Finitely generated abelian groups, invariant factors and elementary divisors, free abelian groups, Smith normal form for matrices, linear independence, homomorphisms of free abelian groups

10. Elementary matrices, symmetry groups with examples, orbits, stabilizers, kernel of an action, orbit stabilizer theorem, Cayley theorem, regular action and conjugacy action

11. Centralizers, conjugacy classes, Burnside formula for orbits, finite subgroups of SO(3)

12. Fixed points of an action, fixed point theorem for p-groups, Cauchy's theorem, even order theorem, groups of order pq, rings, subrings, rings with identity, commutative rings, complex numbers, fields

13. Polynomial rings, division algorithm for polynomials over fields, homomorphisms of rings, unital homomorphisms, isomorphisms and automorphisms of rings, substitution principle for polynomials over rings, kernels of homomorphisms, ideals

14. Generators for ideals, principal ideals, ideals in polynomial rings over fields, quotient rings, homeomorphism and isomorphism theorems for rings, domains, field of fractions, units and irreducibility in domains, Gaussian integers, norm function, associates in domains

15. Primes in domains, principal ideal domains, unique factorization domains, Fermat's theorem for sums of two squares, Dedekind's ideal numbers and the development of ideals

Format

- This course features video lectures from the UIUC Fall 2021 course taught by Professor Charles Rezk and includes online lecture notes via Moodle.

- Text: Goodman, Frederick M. (2014). Algebra: Abstract and Concrete. (Edition 2.6). SemiSimple Press

- Students must be able view assignments online, write out solutions, then scan or take a photo of their written work and upload it to Moodle to meet set deadlines.

- This course requires multiple proctored exams.